



SOME RECOMMENDATIONS FOR CALCULATING FINITE AND INFINITE SUMS

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Abstract: *Calculating finite and infinite sums is one of the important problems of mathematical analysis, in which various methods and theoretical approaches are applied. Articles on this topic are useful not only for students but also for applicants who wish to study this subject in depth. In this article, the main methods of calculating sums are considered through simple examples based on mathematics and computational techniques. General formulas for finite sums, arithmetic and geometric progressions, as well as examples involving recurrent relations are presented. The article is designed to provide practical assistance in understanding the topic and may be useful for the fields of mathematics, physics, and engineering.*

Keywords: *finite sums, infinite series, telescoping sums, summation, limit.*

INTRODUCTION

In mathematical analysis, the theory of sums is considered one of the fundamental concepts, serving as an important methodological basis in the study of the properties of numerical sequences and series. In particular, the investigation of finite and infinite sums is closely connected with algebraic structures, limit processes, and the theory of convergence.

Finite sums are regarded as combinations of a finite number of terms, and various algebraic transformations, recurrent formulas, and methods for obtaining closed-form expressions are applied to them. Infinite sums, on the other hand, constitute the central object of

functional analysis and series theory, and their convergence is determined through the concept of limits.

In this process, the sequence of partial sums, convergence criteria, and various tests play a significant role. At the same time, methods such as telescoping sums, decomposition of series, and regrouping of terms serve as effective tools for simplifying complex expressions.

MAIN PART

The main theoretical and practical aspects of calculating finite and infinite sums are analyzed. In particular, effective methods for evaluating sums are illustrated through examples using algebraic transformations, telescoping



techniques, and recurrent relations. These approaches contribute to a deeper understanding of mathematical analysis and to the development of optimal strategies for solving practical problems.

Definition: A finite sum is the arithmetic addition of a finite number of terms determined by a specific rule or formula.

$$\sum_{k=1}^9 k^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 = 385$$

In calculating finite sums, it is necessary to note the following properties that we need to know.

$$1) \sum_{k=0}^n (\alpha x_k + \beta y_k) = \alpha \sum_{k=1}^n x_k + \beta \sum_{k=1}^n y_k$$

$$2) \text{ If } 1 \leq m < n \text{ then: } \sum_{k=1}^n x_k = \sum_{k=1}^m x_k + \sum_{k=m+1}^n x_k$$

3) The sum of identical terms is equal to the number of terms multiplied by the value of each term. If $X_k = C$ (a constant), then: $\sum_{k=1}^n c = n \cdot c$

Now let us move on to the definitions of infinite sums:

An **infinite sum** is defined as the sum of a sequence of numbers whose terms continue indefinitely.

$$\sum_{k=1}^{\infty} a_n = a_1 + a_2 + \dots$$

n exists and is finite, then the series converges to $\lim_{n \rightarrow \infty} S_n = S$

If this limit does not exist or is infinite, the infinite sum is called **divergent**. At the same time, the properties we presented for finite sums are also applicable to infinite sums.

CONCLUSION. The following examples will help reinforce the knowledge on this topic.

$$1) S_n = 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + n \cdot n!$$

Method 1: Algebraic Transformation. This method involves manipulating the terms of a sum using algebraic operations—such as factoring, expanding, or rearranging terms—to simplify the calculation of finite or infinite sums.

$$\begin{aligned} k \cdot k! &= (k + 1 - 1) \cdot k! = \\ &= (k + 1)! - k! \\ S_n &= \sum_{k=1}^n [(k + 1)! - k!] = \\ &= (2! - 1!) + (3! - 2!) + \dots + \\ &= ((n + 1)! - n) \\ S_n &= 2(n + 1)! - 1 \end{aligned}$$

Method 2: (Difference Method / Telescoping Method). This method is based on expressing terms of a sum as differences of consecutive expressions, so that most terms cancel out when summed, leaving only a few terms to calculate. It is especially useful for simplifying complex series.

$$\begin{aligned} - \left\{ \begin{aligned} S_n &= 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + n \cdot n! \\ S_{n-1} &= 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + (n - 1)(n - 1)! \end{aligned} \right. \\ S_n - S_{n-1} &= n \cdot n! = (n + 1)! - n \cdot n! \end{aligned}$$

$$\begin{aligned} S_1 &= 1 \\ S_n &= S_1 + \sum_{k=2}^n [(k + 1)! - k!] \\ &= 1 + [n + 1]! - k! \\ &= (n + 1)! - 1 \end{aligned}$$

$$\begin{aligned} 2) \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \dots + \frac{1}{n(n+1)(n+2)} \\ a_n = \frac{1}{n(n+1)(n+2)} \\ \frac{1}{n(n+1)(n+2)} = \frac{A}{n} + \frac{B}{n+1} + \frac{C}{n+2} \end{aligned}$$



$$1 = A(n + 1)(n + 2) + B \cdot n(n + 2) + C \cdot n(n + 1)$$

$$\begin{cases} n = 0 \Rightarrow A = \frac{1}{2} \\ n = -1 \Rightarrow B = -1 \\ n = -2 \Rightarrow C = \frac{1}{2} \end{cases} \Rightarrow$$

$$\frac{1}{n(n+1)(n+2)} = \frac{1}{2n} - \frac{1}{2n+1} + \frac{1}{2(n+2)}$$

$$S_n = \sum_{k=1}^n \left(\frac{1}{2k} - \frac{1}{k+1} + \frac{1}{2(k+2)} \right)$$

$$= \frac{1}{2} \sum_{k=1}^n \frac{1}{k} - \sum_{j=2}^{n+1} \frac{1}{j}$$

$$+ \frac{1}{2} \sum_{j=3}^{n+2} \frac{1}{j}$$

$$S_n = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$$

$$3) \frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} +$$

$$\frac{1}{1+2+3+4} + \dots + \frac{1}{1+2+\dots+n} = S_n$$

$$a_n = \frac{1}{1+\dots+n} = \frac{1}{\frac{n(n+1)}{2}} = \frac{2}{n(n+1)}$$

$$= 2 \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

$$\sum_1^n 2 \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

$$= 2 \left(1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \dots + \frac{1}{n} \right)$$

$$- \frac{1}{n+1} = \frac{2n}{n+1}$$

$$S_n = \frac{2n}{n+1}$$

{n} ⇒

The fractional part of a number

[n] ⇒ integer part

$$4) \sum_{n=2}^k \{n - \sqrt{n}\} + \{n + \sqrt{n}\} = ?$$

$$\sum_{n=2}^k \{n - \sqrt{n}\} + \{n + \sqrt{n}\} =$$

$$= \sum_{n=2}^k \{-\sqrt{n}\} + \{+\sqrt{n}\}$$

$$= \sum_{n=2}^k -\sqrt{n} - [-\sqrt{n}]$$

$$+ \sqrt{n} - [\sqrt{n}]$$

$$= \sum_{n=2}^k [-\sqrt{n}] + [\sqrt{n}] =$$

Now we will assign a value to k.

$$k = 2017 - ([-\sqrt{1}] + [\sqrt{1}] + [-\sqrt{2}] + [\sqrt{2}] + \dots + [-\sqrt{2017}] + [\sqrt{2017}]) = -((-1 + 1) + (-2 + 1) + \dots + (-45 + 44)) \Rightarrow$$

$$n = t^2 \text{ va } t \in$$

N in the case of $[-\sqrt{n}] + [\sqrt{n}] = 0$

$n \neq t^2 \text{ va } t \in N$ in the case of $[-\sqrt{n}]$

$$+ [\sqrt{n}] = -1$$

$[\sqrt{2017}] = 44$ thus, in the case of

0, it occurs 44 times.

$2017 - 44 = 1973$ thus, in the case of -

1, it occurs 1973 times.

$$\sum_{n=2}^{2017} \{n - \sqrt{n}\} + \{n + \sqrt{n}\} = 1973$$

Let us present another very simple example. We are asked to find the sum of the first nnn odd numbers.

$$1 + 3 + 5 + \dots + (2n - 1) = \sum_{k=1}^n (2k - 1)$$

Solution:

$$\sum_{k=1}^n (2k - 1) = \sum_{k=1}^n (k^2 - (k - 1)^2) = n^2$$

This gives us a wonderful formula: the sum of the first nnn positive odd numbers is equal to n^2 . To perform this calculation, it is sufficient to know the following simple algebraic formula:

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$



In particular, we will need $(k - 1)^2 = k^2 - 2k + 1$ but we use it in the form: $2k - 1 = k^2 - (k - 1)^2$

As a result of the analysis, it has been determined that the choice of an appropriate method for computing sums directly depends on the structural properties of the problem, which significantly increases computational efficiency. In particular, the telescoping method and recursive approaches provide optimal solutions for certain classes of problems.

Let us consider the solution of another example. Calculate:

$$1^2 \cdot 2! + 2^2 \cdot 3! + 3^2 \cdot 4! + \dots + n^2 \cdot (n + 1)!$$

Solution: We write the expression in the form $k^2 \cdot (k + 1)! = a_{k+1} - a_k$ ($1 \leq k \leq n$) and try to determine a_k . We look for a_k in the form $a_k = k! b_k$, where b_k — is some number. In this case, the relation above takes the simpler form: $k^2 \cdot (k + 1)! = (k + 1) \cdot b_{k+1} - b_k$

Next, we assume $b_k = P(k)$ where $P(k)$ is a polynomial. Then we get:

$$k^2 \cdot (k + 1) = (k + 1) \cdot P(k + 1) - P(k)$$

For this identity to hold for all natural numbers k the $P(k)$ must have the form

$$P(x) = x^2 + cx + d$$

Substituting this into the previous relation, we obtain:

$$k^2 \cdot (k + 1) = (k + 1)((k + 1)^2 + c(k + 1) + d) - k^2 - ck - d$$

From the resulting identity, we can easily find $c = -1$ and $d = -2$.

Thus:

$$k^2 \cdot (k + 1) = (k + 1)((k + 1)^2 - (k + 1) - 2) - k!(k^2 - k - 2)$$

From this, we obtain:

$$\begin{aligned} \sum_{k=1}^n k^2 \cdot (k + 1)! &= \sum_{k=1}^n [(k + 1)!((k + 1)^2 - (k + 1) - 2) - k!(k^2 - k - 2)] = \\ &= (n + 2)(n - 1)(n + 1)! + 2 = (n - 1)(n + 2)! + 2 \end{aligned}$$

CONCLUSION

In this study, theoretical approaches and practical methods for calculating finite and infinite sums were systematically analyzed. In particular, the effectiveness of algebraic transformations, telescoping sums, recurrence relations, and series decomposition methods was demonstrated through various examples. It was shown that, using these methods, it is possible to express complex sums in closed-form formulas. Additionally, in the study of infinite sums, the issue of convergence plays a crucial role. In this context, the existence of a limit for the sequence of partial sums is considered the main criterion. These theoretical foundations serve as an important basis for mastering further topics in mathematical analysis.

Overall, the results of this research indicate that a deep understanding of methods for evaluating finite and infinite sums is essential not only for strengthening theoretical knowledge but



also for effectively solving practical and applied problems.

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